Metacomputation as a Tool for Formal Linguistic Modeling

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Abstract

We consider the principle “a new model is a model of an existing one” as the main scheme for deriving new linguistic models by metacomputation. We derive the basic requirements for metacomputation by a structural analysis of different model definitions, and show that in order to automate the creation of linguistic models the following operations on linguistic models have to be performed by metacomputation effectively and efficiently: composition, inversion, and specialization of algorithms. This may also serve as a unifying paradigm for different program transformation approaches.

1. INTRODUCTION

During the last decades we have witnessed tremendous technological breakthroughs in the development and application of computers. The introduction of the computer was an evolutionary step in the control of formal linguistic models, a metasystem transition (MST). As a result the number of linguistic models created and used has significantly increased.

The method of modern science is, in its essence, the creation of linguistic models [1]. Informally, a model is a process which somehow mimics, or simulates, another primary process. Using a model it becomes possible for a system $S$ to predict and know something about the primary process before it actually happens, or without performing it. As does every branch of science, computer science has its own types of objects, namely formal linguistic models, and the models of the models computer science creates itself. Linguistic models that can be executed on a computer, at least in principle, are referred to as algorithms, or programs.

The computer was really necessary before one could start to learn more about formal linguistic modeling on a large scale (human beings are neither precise enough, nor fast enough to carry out any but the simplest procedures). Just as mastering the general principle of using tools gives rise to the creation of industrial systems, mastering the principle of linguistic modeling gives rise to the creation of hierarchical systems of formal languages (on which modern science is based).

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The next evolutionary step in linguistic modeling is the control over the creation of formal linguistic models by the computer. In this paper we consider the principle “a new model is a model of an existing one” as the main scheme for the derivation of new models. This is performed by a process referred to as metacomputation. This viewpoint determines which operations metacomputation has to perform efficiently on formal linguistic models in order to evolve linguistic modeling: composition, inversion, and specialization of algorithms. Indeed, many known problems in computer science can be placed in these categories.

2. LINGUISTIC MODELING AND METACOMPUTATION

In this paper, two aspects of linguistic modeling are studied (in both cases, control means the automation / mechanization of the respective activity):

control of executing models = computation
control of creating models = metacomputation

The use of the computer to execute models was the first step in controlling linguistic modeling, but it is not the last step. Indeed, the creation of linguistic models was not directly affected by the introduction of the computer. At the beginning, this activity was fully performed by the human (‘programming’). Later, computer science has developed various methods to achieve more control over these activities, such as the development of new language paradigms, the construction of tools for generating special-purpose programs (e.g., scanners, parsers), and a variety of approaches for the verification, transformation, and compilation of programs. However, the basic problem still exists: how are we to achieve control over the creation of linguistic models, in general, and of programs, in particular?

Our approach is to use the method of linguistic modeling itself. Metacomputation is the creation of new models from existing models. This term underlines the fact that model creation is one metasystem level higher than computation. From now on we will refer to formal linguistic models as linguistic models, or simply as models.

Modeling The common notion of modeling, often referred to as the modeling scheme, is as follows (Fig. 1). Let an object \( o \) be in some state, which is characterized by the information \( x_o \), and suppose the object performs an action. We shall denote by \( y_o = F_o(x_o) \) the information about the ensuing state.\(^3\) Suppose we want to make a prediction about \( y_o \). Modeling introduces another object \( m \), a model, for making predictions about the object \( o \), considering the model ‘similar’, in some sense, to the object. A model \( m \) is an abstraction of the object \( o \), that is, a model contains less information than the object.[1]

The mappings \( H_{in} \) and \( H_{out} \) are two abstraction functions, often referred to as homomorphisms. They map the information \( x_o, y_o \) about the object \( o \) to the information \( x_m, y_m \) in the model \( m \) (since we consider only linguistic models, we may safely assume that the functions \( H_{in}, H_{out} \) do not interfere with the behavior of the object \( o \)). Having full information \( x_o, y_o \), we can deduce the corresponding information \( x_m, y_m \) in the model, but

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\(^3\) We do not assume that these actions are deterministic. \( F_o(x_o) \) denotes any of the possible states after some action. We can think of \( F_o \) as a non-deterministic function, or a relation.
not vice versa. Having information $y_m$, we cannot, in general, predict a precise fact $y_o$ about the object, but only that $H_{out}(y_o) = y_m$. That is, the possible outcome belongs to a set $\{y_o \mid H_{out}(y_o) = y_m\}$. Thus by using $F_m$ one can predict, to some extent, the state of the object resulting from $F_o$:

$$y_m = F_m(H_{in}(x_o)) = H_{out}(F_o(x_o))$$

**Creating models from models** Let the object $o$ itself be a model and assume that we want to create another model $m$ being a model of the first model $o$. Initially we are provided with the object $o$: that is, a description of the domains $X_o$ and $Y_o$ over which $x_o$ and $y_o$ range, and a description of the function $F_o: X_o \rightarrow Y_o$. We need to define the domains $X_m$ and $Y_m$ which $x_m$ and $y_m$ range over, and the function $F_m: X_m \rightarrow Y_m$ of the derived model $m$.

What can be automated in the creation of a new model? The choice of the information available for building the new model is a creative step that depends on the external goals of the user. Consequently, we will not address the problem of how to choose homomorphisms $H_{in}$ and $H_{out}$; this choice will be left to the user.

The main task is to automate the construction of the new function $F_m$. From the modeling scheme (Fig. 1) we see that $F_m$ can be defined by using the mappings $H_{in}^{-1}$ and $H_{out}$ (where $H_{in}^{-1}$ is an inverse of $H_{in}$). A full definition of $F_m$ is provided by $F_o$, $H_{in}^{-1}$ and $H_{out}$:

$$\text{def } F_m(x_m) = H_{out}(F_o(H_{in}^{-1}(x_m)))$$

**Metacomputation** The goal of metacomputation is to derive new models from such formal definitions. Let us denote the process of performing metacomputation by $Mc$. In order to express that metacomputation is applied to the textual definition of the model rather than to its denotation, we move the expression downwards (filling the remaining space with a line):

$$Mc(__________________) \Rightarrow F_m$$

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4 To distinguish definitions from statements and equations, we use the keyword `def`. The function on the left hand side (e.g. $F_m$) is defined by the expression on the right-hand side.
We refer to such a formula as *MST-formula* because it describes the activity of creating models, which is a meta-activity as compared with just executing them. The essence of metacomputation is considering models as material that can be transformed and manipulated in various ways.

**Requirements for metacomputation** Two operations are involved in this formula:

- **composition**: $F_o$ composed with $H_{in}^{-1}$, $H_{out}$ composed with $F_o$
- **inversion**: $H_{in}^{-1}$

If metacomputation is capable of deriving efficient models defined by these operations, then the creation of formal linguistic models is automated to a large extent. In other words, performing these operations effectively and efficiently is a prerequisite for a successful application of metacomputation.

3. **SELECTED PROBLEMS OF METACOMPUTATION**

3.1 **Problem of Program Composition**

Consider the case when the information $x_o$ on the object model $o$ is identical to the information $x_m$ on the new model $m$ (Fig. 2). That is, the homomorphism $H_{in}$ is the identity function: $x_m = H_{in}(x_o) = x_o$. The function of the new model is then defined as:

$$ F_m(x_m) = H_{out}(F_o(x_m)) $$

Assume that we are interested in a small part of the output $y_o$. To define the new model, we provide the homomorphism $H_{out}$ selecting the parts of $y_o$ we are interested in. However, directly computing the above definition of the new model does not decrease the amount of computer resources needed to obtain the result $y_m$. In this case it might pay off to create a new model by metacomputation:

$$ Mc(\quad) \Rightarrow F_m $$

The new model may be more efficient and drastically reduced in size. One would expect that redundant computations in $F_o$ are removed during metacomputation, and only those computations that are needed to produce the information selected by $H_{out}$ are present in $F_m$. This problem has been studied in connection with program slicing [2].

**Example** A particular application of the metacomputation of composition is deriving an efficient interpreter from a compiler. *Compilation* is the process of translating programs from one language, say L, to another language, say M, where $p_L$ and $p_M$ are programs written in L and M respectively:

$$ CompLM(p_L) \Rightarrow p_M $$

*Interpretation* is performing the activity implied by a program, say $p_M$, using another, universal program, say $Int_M$, called the interpreter (below $x$ is the initial information used in $p_M$, and $y$ is the result):

$$ IntM(p_M, x) \Rightarrow y $$
Assume that an \( L \rightarrow M \)-compiler \( \text{CompLM} \) and an \( M \)-interpreter \( \text{IntM} \) are given. Then we can execute a program \( p_L \) in two stages: first by translating \( p_L \) into \( M \), and then by interpreting the \( M \)-program:

\[
\text{IntM} (\text{CompLM} (p_L), x) \Rightarrow y
\]

That is, a new interpreter, \( \text{IntL} \), may be defined by the composition of \( \text{CompML} \) and \( \text{IntM} \):

\[
\text{def } \text{IntL} (p, x) = \text{IntM} (\text{CompLM} (p), x)
\]

However, the step via the intermediate language \( M \) may be rather inefficient. If we meta-compute this definition, we may obtain a more efficient interpreter.

\[
\text{Mc}(\text{IntM} (\text{CompLM} (p), x)) \Rightarrow \text{IntL}
\]

This is a metasystem transition over the computation process defined by the composition. The importance of effectively metacomputing the composition of models is hard to overestimate, since composition is one of the basic methods for building new models from existing ones.

### 3.2 Problem of Program Inversion

Consider the case of deriving a new model when the output information \( y_o \) on the object model is identical to the output information \( y_m \) on the new model (Fig. 3). That is, the homomorphism \( H_{\text{out}} \) is the identity function: \( y_m = H_{\text{out}} (y_o) = y_o \). The function \( F_m \) of the model is then defined as follows:

\[
\text{def } F_m (x_m) = F_o (H_{\text{in}}^{-1} (x_m))
\]

As in the case of composition, one may derive a new model \( F_m \) by metacomputation:

\[
\text{Mc}(F_o (H_{\text{in}}^{-1} (x_m))) \Rightarrow F_m
\]

In this case a combination of composition and inversion is used. We want to derive a new model that can be used to make a prediction \( y_m \) using the partial information \( x_m \) about \( x_o \). The homomorphism \( H_{\text{in}} \) defines what is to be known about \( x_o \): \( x_m = H_{\text{in}} (x_o) \). In some instances, the partial information \( x_m \) will be sufficient to produce \( y_m \). Then \( F_m (x) \) should
return it. However, generally $x_m$ does not define $y_m$ precisely. In this case, one is interested either in one of the possible results, or in the set of all possible results. These two choices correspond to two different kinds of inversions.

The problem of constructing an inverse relation is interesting in its own right, since many mathematical problems are stated in the following way: given the description $P$ of some properties of objects, find (at least one) object $x$ such that $P(x)$ holds. This is referred to as the inversion problem. The predicate $P$ may be formulated as an algorithm checking the linguistic object $x$.

The inversion of programs is a fundamental problem, and a large branch of computer science has been based on solutions emerging from logic and proof theory [3,4]. Direct methods for inverting algorithms have been developed [5-7]. By varying the metaevaluator $Mc$ and the method for solving the inverse problem different linguistic models can be generated by MST-formulas involving composition and inversion.

3.3 Problem of Program Specialization

Automatic inversion by metacomputation is a hard problem, and hence it is important to consider variants of the modeling scheme without inversion. This leads us to the problem of specialization of models.

The relation of $x_o$ and $x_m$ may be established not only by a homomorphism from $x_o$ to $x_m$, but by a mapping, say $G$, from some information $x_m$ given in the model, to $x_o$. That is, the inverse mapping $G(x_m) = H_{in}^{-1}(x_m)$ is supplied by the user. This corresponds to changing the direction of the arrow marked with $H_{in}$ (Fig. 3). The function $F_m$ of the new model is then defined by:

$$\text{def } F_m(x_m) = F_o(G(x_m))$$

The problem in this definition has the same structure as the problem of composition. However, since the modeling function $F_o$ is usually much more complex than the mappings $H_{out}$ and $G$, there is a difference between the two cases: in Section 3.1, the outer function is simpler, and here, the inner function is simpler. Different methods of metacomputation may be advantageous in each case. The problem of specialization falls into the second case.
The problem of specialization arises when the domain for which a model is used is restricted (Fig. 4). The function $F_s$ of the specialized model is the same as the function $F_o$ of the model $o$, but the input ranges only over a part of the domain of model $o$:

$$\text{def } F_s(x) = F_o(x) \text{ if } x \in S,$$

where $S$ is a subset of the domain of model $o$.

Although the definitions of $F_s$ and $F_m$ above are not formally equivalent, in the majority of the cases, the definition of $F_s$ can be replaced by one, where the mapping $G(x_m)$ is a representation of the set $S$: $S = \{G(x_m) \mid x_m \in X_m\}$.

**Example** Consider a model whose function $F_o$ has several arguments, such as $F_o(x_1, x_2)$ (formally it takes a tuple). Then the mapping $G$ may return a tuple in which parts of the arguments are fixed to some information. Assume that $x_1$ is always mapped to the same information $I$. Then $x_2$ is the remaining parameter, and $G(x_2) = (I, x_2)$:

$$\text{def } F_m(x_2) = F_o(I, x_2)$$

As usually, one may derive a more efficient model $F_m$ by metacomputing the definition:

$$\text{Mc}(\text{________}) \Rightarrow F_m$$

The motivation to metacompute the definition of a model whose input domain is restricted is to remove redundant computations that may be present in the object model but are not necessary for the narrowed domain. This can give substantial savings, e.g., when one parameter, say $x_1$, changes less frequently than another.

Surprisingly, many problems in computer science reduce to the problem of specialization, including the central problem of metacomputation: the problem of self-applying metacomputation. It was found [8] that the solution to the problem of generating compilers from interpreters requires neither composition nor inversion of programs, just fixing some of the arguments is sufficient (as in the example above).

Although the problem of specialization is a special case of the problem of composition, it is worth considering it separately because of the large number of practical problems that require specialization. This gave rise to a new branch in computer science, called partial evaluation, which has been developing rapidly during the past decade due to advances achieved both in theory and practice [9].
4. Conclusion

Computer science, as compared to other disciplines, appears as a rather diverse field lacking a clear focus and a notion of what has to be achieved. In this contribution we tried to emphasize common roots of different problems. We discussed how the notion of linguistic modeling may serve as a unifying viewpoint and derived the requirements for metacomputation by a structural analysis of the problem. We saw that to solve the problem of linguistic modeling, metacomputation must perform the following operations efficiently: composition, inversion, and specialization (the latter being a special, though important, case of composition). This task may serve as a clear guideline for research in metacomputation.

The goal is to make the automatic derivation of models by metacomputation a practical tool. Since the 60s many solutions have been tried and some progress has been achieved, but the basic problems still remain open. We say that the next evolutionary step in formal linguistic modeling, the next large-scale metasystem transition, is achieved if efficient linguistic models can be created by the computer and it suffices for the human to make initial formal definitions. The ultimate goal is to achieve the ability for an arbitrary series of metacomputations over linguistic models to be just an ordinary, mechanical process. In this sense, we are actually working towards the next metasystem transition in linguistic modeling. We believe that this is one of the most challenging tasks of computer science.

References


— Dedicated to the memory of Alexander Romanenko —